Quantifying the Effects of Patent Protection on Innovation, Imitation, Growth, and Aggregate Productivity†

Pedro Bento
Texas A&M University*

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ABSTRACT

I develop a general equilibrium model in which patent protection can increase or decrease the costs of sequential innovation, original innovation, and imitation. Depending on these relative effects, protection can in theory increase or decrease markups, imitation, innovation, growth, and aggregate productivity. I discipline the model using data from several different sources, and find that weakening protection in the U.S. would lead to no change in markups and imitation, no change in long-run growth, a more than doubling of the number of firms, and an increase in aggregate productivity of 9 percent.

Keywords: patent protection, firm size, productivity, innovation, imitation, competition.
JEL codes: O1, O3, O4.

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*Texas A&M University, Department of Economics, 3056 Allen Building, 4228 TAMU, College Station, TX 77843. E-mail: pbento@tamu.edu.
1 Introduction

Does patent protection encourage innovation? If so, through what mechanisms? Patent protection is generally thought to encourage innovation by discouraging imitation, and several theoretical papers have investigated the optimal trade-off between the level of innovation and the prices faced by consumers. But empirical studies have been hard-pressed to find a robust positive relationship between protection and innovation,\(^1\) and evidence of any relationship between patent protection and imitation or markups has been surprisingly absent. Regardless of how patent protection affects imitation or aggregate innovation, the allocation of innovative inputs may still depend on the strength of patent protection. For example, Chu, Cozzi, and Galli (2012) show that if patent protection transfers royalties from sequential innovators (who attempt to improve on patent-protected products) to original innovators, then stronger protection may induce a reallocation from sequential to original innovation. Although growth may decrease due to slower quality improvements, welfare may still be higher due to an increase in the number of varieties available to consumers.

In this paper I develop a general equilibrium model that allows for patent protection to work through all of these mechanisms, without any restrictive assumptions about the qualitative effects of patent protection on original innovation, sequential innovation, or imitation. In the model, innovative firms choose between creating original products (thereby creating new product markets) or improved versions of existing products. Successful innovators enjoy monopoly profits at first, but then face competition from imitators. Eventually, a new round of sequential innovation occurs in each market, and an incumbent may either retain its incumbent status or lose the market to a new innovator. Product lines become obsolete with some probability, so there is a continuous churning of product markets along the balanced growth path of the economy. I model patent protection in a way that captures many mechanisms present in the existing

\(^1\)For example, all of the following papers report either less innovation or no effect on innovation from stronger patent protection: Sakakibara and Branstetter (2001); Qian (2007); Boldrin and Levine (2008, 2013); Lerner (2009); Moser and Voena (2012); Moser (2013).
literature. In the model, patent protection affects the costs of imitating and innovating, but may affect each differently. For example for certain parameter values, protection may increase costs for sequential innovators (perhaps due to the cost of working around previous patents) but decrease costs for original innovators, as in Chu, Cozzi, and Galli (2012). Under different parameter values, protection may reduce the cost of all innovation (perhaps due to a quicker diffusion of new ideas), but increase the cost of imitation, as in Helpman (1993). Differences in the absolute and relative effects of patent protection on innovation and imitation generate different outcomes for markups, growth, aggregate productivity, and the number of firms in the economy. By capturing each of these mechanisms in a general equilibrium model, I reveal important interactions between innovators and imitators. For example if patent protection increases the cost of imitating a protected innovation (say, by increasing the cost of working around the patent), then imitation may still increase if protection increases the cost of innovation more than proportionately. In this particular case, the number of original innovations will decrease and reduce the equilibrium number of product markets. This reduction in the number of markets reduces the demand for labor, which can reduce the wage enough to more than offset the higher cost of imitation, and thereby encourage more imitation in each product market.

To discipline the parameter values in the model and gauge the impact of patent protection in practice, I take the model to the data. I obtain information about the relative effects of protection on innovators and imitators by using the model to interpret empirical relationships between the strength of patent protection and markups, the number of firms, and the ratio of original to sequential innovations. The data suggests that there is no significant relationship between the strength of patent protection and markups, nor between protection and the ratio of original to sequential innovators. Interpreted through the model, these empirical results suggest that protection affects the costs of imitators and all innovators proportionately. This implies that stronger protection should not affect long-run growth or markups, as the entire adjustment to changes in the strength of protection must work through original innovation. If the costs
of innovation and imitation *increase* with protection, then aggregate innovation, the variety of products, and the number of firms must all decrease with protection. This decreases the potential revenue from each product market just enough to leave the value of entry constant for imitators and sequential innovators. If stronger patent protection *decreases* the costs of innovation and imitation, then innovation, variety, and the number of firms must all increase. Using recently constructed data on the number of manufacturing firms across countries from Bento and Restuccia (2016), I document evidence that the number of firms is in fact decreasing in patent protection. Using the calibrated model to quantify the effects of patent protection, I estimate that weakening protection in the U.S. would lead to a 77 percent increase in the number of firms, and an *increase* in aggregate productivity of 9 percent, without affecting markups or the long-run growth rate.

Aspects of the model developed in this paper are similar to Chu, Cozzi, and Galli (2012), who model patent protection as a transfer of revenue from sequential innovators to original innovators in a general equilibrium setting, but the present model differs in three important ways. First, the model is cast in discrete time, which more easily allows for an endogenous number of firms (as in Bento, 2014).² This allows me to exploit data on the number of firms across countries (Bento and Restuccia, 2016). Second, I allow for imitation, generating testable implications for markups. Third, I allow for the possibility that patent protection affects the costs of all potential patent infringers, as they may need to use up resources to work around any relevant patents. Modeling patent protection as affecting the costs of innovation and imitation rather than as a transfer of profits (as Chu, Cozzi, and Gali do, for example) is consistent with the findings of both Mansfield, Schwartz, and Wagner (1981) and Zander and Kogut (1995), who report that patents are virtually never licensed to potential competitors.

Scotchmer (1991) analyzes optimal patent policy when innovation is sequential, as do a large number of subsequent papers like O’Donoghue and Zweimüller (2004), Hopenhayn, Llobet, and

²Chu, Cozzi, and Gali (2012) extend Grossman and Helpman (1991), and as a result the number of innovating firms in their model is indeterminate.
Mitchell (2006), Bessen and Maskin (2009), and Acemoglu and Akcigit (2012). In each of these papers the number of innovators is either fixed or indeterminate, and imitation is neglected. Segerstrom (1991) and Helpman (1993) consider imitation in models of sequential innovation, but do not account for different types of innovation as I do here. As extensions of Grossman and Helpman (1991), their models are also silent about the determinants of the number of firms.

By quantifying the effects of patent protection on the number of firms, this paper contributes to the growing literature attempting to explain differences in firm size (the inverse of the number of firms per worker) across countries. See, for example, Bhattacharya, Guner, and Ventura (2013), Hsieh and Klenow (2014), and Bento and Restuccia (2016), who analyse the effects of misallocation on firm size and other outcomes.

Another contribution of this paper is in developing a model with endogenous markups where both markups and the number of firms are independent of population. This is consistent with the findings of Bento and Restuccia (2016), but in direct contrast to all current models with endogenous markups and endogenous firm entry.3

In the next section I describe the model, characterize equilibrium, and discuss its key implications. In Section 3 I introduce and use data from several sources to calibrate the model and show that weakening patent protection in the U.S. would increase productivity. I also discuss how my quantitative results are robust to alternative assumptions in the model. The final section concludes.

3As far as I know. For example, see Melitz and Ottaviano (2008), Desmet and Parente (2010), Peters (2013), and Edmond, Midrigan, and Xu (2015). In each of these models a larger population increases the size of each market, which encourages entry. More entry reduces markups which stops the number of firms from increasing proportionately with population. As a result, these models imply that the number of firms per worker should be lower in more populous economies.
2 Model

Consider an economy in which time is discrete and a final good is produced using a variety of inputs from a representative intermediate industry. Intermediate firms (hereafter referred to as ‘firms’) produce these inputs one-for-one using labor. The final good can be used both for consumption and to finance the introduction of new products, and will also act as the numéraire. There are a large number of potential innovating firms, any of which can choose to introduce an improved version of an existing product or create an original product. There are also a large number of potential imitators, any of which can choose to create an exact copy of an existing product. I assume products can be imitated after one period, but improved only after two periods. In addition, each product market faces an exogenous probability of destruction every two periods (before each new round of innovation), so new product markets (spawned by original products) continue to be created along the balanced growth path (BGP). I study the stationary decentralized equilibrium of the economy along a BGP, in which firms take the economy-wide wage, growth rate, and interest rate as given, and free entry ensures the value of entry for all innovators and imitators is driven to zero. I begin by describing the environment in more detail.

2.1 Environment

There is a continuum of identical consumers of constant measure $L$, each supplying one unit of labor to intermediate firms. Each consumer values only consumption ($C$) and has a constant discount factor $\beta \in (0, 1)$. Preferences over the stream of consumption in each period are

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4Given the lack of capital in the model, output per capita will be equivalent to aggregate productivity. I therefore use ‘output per capita’ and ‘aggregate productivity’ interchangeably.

5This is consistent with Mansfield, Schwartz, and Wagner (1981) and Zander and Kogut (1995), who find that imitating a product takes just over half the time required to create the product in the first place (on average).
described by the following log-utility function;

\[ \sum_{t=0}^{\infty} \beta^t \log(C_t). \]

The market for final output is perfectly competitive, with a representative firm using inputs from a representative intermediate industry to produce output according to the following production function;\(^6\)

\[ Y = \left( \int_0^M \left( \sum_{n=1}^{N_m} A_{n,m}^{\frac{1-\alpha}{\alpha}} y_{n,m} \right)^{\alpha} \right)^{\frac{1}{\alpha}}, \]

where \( m \) indexes a continuum of product markets of measure \( M \), \( y_{n,m} \) is the quantity of input \( m \) demanded from firm \( n \), \( A_{n,m} \) is the quality of input \( m \) produced by firm \( n \), and \( \frac{1}{1-\alpha} > 1 \) is the constant elasticity of substitution between differentiated product markets.

Different products within a product market are perfectly substitutable, and I make the following two assumptions with respect to firms’ price and quantity decisions. First, if multiple firms produce an input using the same quality \( A_m \), then they engage in cournot competition, choosing output while taking the output of competitors as given. Second, if firms are heterogeneous with respect to quality, then the firm with the 1st-best quality \( A_{n,m} = A_{[1],m} \) can commit to charging a limit price if any rivals attempt to produce, ensuring only the best firm in the market produces in equilibrium. These assumptions about market structure are made solely for the sake of exposition. Although they can obviously play a part in determining the effects of patent protection in theory, in Section 3.2 I show that they play no part in determining the quantitative effects of patent protection in practice.

Finally, I assume each product market is destroyed with probability \( \lambda \) every two periods (before each round of innovation).

\(^6\)Throughout the paper, I omit the time subscript unless clarity requires it.
2.2 Innovation

2.2.1 Sequential Innovations

At the beginning of each period, after any exogenous market death has occurred, potential innovators can choose to undertake the investment necessary to create and introduce an improved product into any market (as long as the current incumbent’s product has existed for two periods). Denote the 1st-best quality in period $t - 2$ in market $m$ by $A_{[1],m,t-2}$. Any firm $i$ that chooses to incur the cost of introducing an improved version of product $m$ in period $t$ receives a quality equal to $A_{i,m,t} = A_{[1],m,t-2} \cdot (1 + h_{i,m,t})$, where $h_i \in (0, \bar{h})$ is randomly drawn from a bounded distribution. There is a fixed cost associated with creating and introducing an improved product, which must be incurred before $h$ is realized. For the incumbent, this fixed cost is equal to $\frac{Y_L}{L} \cdot \Psi_{m,t-2} \cdot c_s' > 0$. For all other sequential innovators, the cost of creating and introducing an improved product is $\frac{Y_L}{L} \cdot \Psi_{m,t-2} \cdot c_s' \cdot \rho^\theta$, $c_s > c_s'$, $\theta \in \mathbb{R}$. $\Psi_{m,t}$ is defined as;

$$
\Psi_{m,t-2} \equiv \frac{A_{[1],m,t-2}}{A_{inn}[1,t-2]},
$$

$$
\frac{A_{inn}[1,t-2]}{A_{[1],t-2}} \equiv \frac{1}{M_{t-2}} \int_{0}^{M_{t-2}^{inn}} A_{[1],m_{inn},t-2} dm_{inn},
$$

where $M_{t-2}^{inn} \subset M_{t-2}$ is the subset of markets in which innovation occurred in period $t - 2$, indexed by $m_{inn}$. Having the cost of innovation scale up with aggregate output per capita is consistent with the findings of Bollard, Klenow, and Li (2016). Making innovation more costly in higher-quality markets is intuitive, and also serves to ensure that innovating firms face the same decision in different markets. The scalar $\rho \geq 1$ represents the strength of patent protection. That $\theta$ can be positive or negative allows for possibility that protection may increase or decrease the cost of sequential innovation.
2.2.2 Original Innovations

At the beginning of each period, potential innovators can also create an original product, thereby creating a new product market. I assume that original innovators must incur a cost of \( Y_L \cdot c_o \cdot \rho^\phi \), \( c_o > 0 \), \( \phi \in \mathbb{R} \), where \( \phi > \theta \) implies that patent protection increases the cost of original innovation relative to sequential innovation (or reduces it less), and \( \phi < \theta \) implies the opposite. When an original product is created, it acquires a quality \( A_o \) equal to the average quality across producers in innovating markets \( \overline{A}_{[1],t} \).

2.3 Imitation

At the beginning of each period, potential imitators may choose to imitate an incumbent’s product and compete in the same market (as long as the incumbent’s product has existed for at least one period). To introduce a copy of the best firm’s product in market \( m \) in period \( t \), an imitator must incur a fixed cost equal to \( Y_L \cdot \Psi_{m,t-1} \cdot \hat{c} \cdot \rho^\gamma \), \( \hat{c} > 0 \), \( \gamma \in \mathbb{R} \), where \( \Psi_{m,t-1} \) and \( \rho \) are defined as above. \( \gamma > 0 \) implies that patent protection increases the cost of imitation (and vice versa), \( \gamma > \theta \) implies that the cost of imitation increases more (or is reduced less) than the cost of sequential innovation, and \( \gamma < \phi \) implies the opposite. An imitator receives the same quality as the incumbent.

2.4 Equilibrium

I focus on the stationary decentralized equilibrium of the model, and make two simplifying assumptions about the economy at time zero. First, I assume markets that initially feature imitation outnumber markets with sequential innovations by a factor of \( 1/(1-\lambda) \). This ensures that in every period, after new products have been created, half of all markets will feature imitation. Second, I assume that initial sequential innovators improve on \( A_{[1],m,-2} = 1/(1 + \)
\( \mathbb{E}[h_{[1]}]^{1/2} \), while initial imitators have a quality equal to \( A_{[1],m,0} = 1 \), where \( \mathbb{E}[h_{[1]}] \) is the average quality improvement across markets with sequential innovation on the balanced growth path. Together, these assumptions ensure that the growth rate is constant from period to period. In such an equilibrium, given these assumptions, the interest rate \( r \) and growth rate of aggregate output are constant, as is the measure of product markets \( M \) (since the mass of new product markets will be equal to the mass of markets becoming obsolete each period), and the wage \( w \) grows at the same rate as aggregate output. In addition, the assumptions made about the environment above will ensure a constant number of both innovators \( N^{inn} \) and imitators \( N^{im} \) per market over time. I begin by describing the decision problems of each agent, and then define and solve for the stationary equilibrium.

2.4.1 Consumer

In each period consumers choose both consumption and savings, and the only vehicle for savings is the purchase of equity in innovating firms, earning a rate of return of \( r \).\(^7\) The representative consumer’s problem in each period \( t' \) is therefore to choose consumption \( C \) and savings \( S \) for each period \( t \geq t' \), given \( w, r, \) and \( g \), to maximize;

\[
\sum_{t=t'}^{\infty} \beta^t \log(C_t), \text{ s.t. } C_t + S_t \leq w_{t'}(1 + g)^{t-t'} + S_{t-1}(1 + r).
\]

The first order conditions for this problem imply the following interest rate;

\[
r = \frac{1 + g}{\beta} - 1.
\]

\(^7\)I take it as given that each consumer will diversify across all innovating firms, as every firm shares the same expected value before innovating. The notation here also takes for granted that \( r \) and \( g \) are constant along the BGP.
2.4.2 Final-Good Producer

In each period, the final-good producer takes the prices of all intermediate inputs as given, and demands inputs from each intermediate product market to maximize profits:

\[ Y - \int_0^M \sum_{n=1}^{N_m} P_{n,m} y_{n,m} \, dm, \]

where \( Y = \left( \int_0^M \left( \sum_{n=1}^{N_m} A_{n,m}^{1-\alpha} y_{n,m} \right)^{\alpha} \, dm \right)^{\frac{1}{\alpha}} \) and \( P_{n,m} \) is the price of input \( m \) from firm \( n \). The first order conditions for the final-good firm’s problem imply the following inverted demand function for markets in which imitation has not yet taken place:

\[ P_{[1],m} = Y^{1-\alpha} A_{[1],m}^{1-\alpha} y_{[1],m}^{\alpha-1}, \]

and the following inverted demand function after imitators have entered a market:

\[ \hat{P}_m = Y^{1-\alpha} A_{[1],m}^{1-\alpha} \left( \hat{y}_{[1],m} + \sum_{n=1}^{N_{im}^m} \hat{y}_{n,m} \right)^{\alpha-1}, \]

where \( N_{im}^m \) is the number of imitators in market \( m \) and \( \hat{y}_{n,m} \) is the output of imitator \( n \).

2.4.3 Intermediate Firms

In markets where sequential innovations are introduced, all but the highest-quality firm in each product market will choose not to produce. Producers face the downward-sloping demand curves implied by the final-good firm’s problem above, and demand labor \( y_{[1],m} \) given the wage \( w \), to maximize operating profits:

\[ \pi_{[1],m} = P_{[1],m} y_{[1],m} - w y_{[1],m}. \]
First order conditions imply an optimal price equal to;

\[ P_{[1],m} = \frac{w}{\alpha}, \]  

\( (1) \)

and optimal output equal to;

\[ y_{[1],m} = YA_{[1],m} \left( \frac{\alpha}{w} \right)^{1-\alpha}. \]

Together, these imply an innovator with quality \( A_{[1],m} \) earns operating profits for one period equal to;

\[ \pi_{[1],m} = YA_{[1],m}(1 - \alpha) \left( \frac{\alpha}{w} \right)^{1-\alpha}. \]

Creators of original products earn similar profits for one period, equal to;

\[ \pi_o = YA_o(1 - \alpha) \left( \frac{\alpha}{w} \right)^{1-\alpha}. \]

After the first period of production, the best firm in market \( m \) (or the only firm, in the case of new markets) must compete with \( N_{im}^m \) imitators. Taking the output of imitators as given, the incumbent firm chooses \( \hat{y}_{[1],m} \) to maximize operating profits;

\[ \hat{\pi}_{[1],m} = \hat{P}_m \hat{y}_{[1],m} - w \hat{y}_{[1],m} \]

\[ = Y^{1-\alpha} A_{[1],m}^{1-\alpha} \left( \hat{y}_{[1],m} + \sum_{n=1}^{N_{im}^m} \hat{y}_{n,m} \right)^{\alpha-1} \hat{y}_{[1],m} - w \hat{y}_{[1],m}. \]

Taking for granted the fact that imitators face the same problem as the best firm, first order conditions imply the following optimal output for each competitor;

\[ \hat{y}_m = YA_{[1],m}(N_{im}^m + \alpha)^{\frac{1}{1-\alpha}} \]

\[ w^{\frac{1}{1-\alpha}} (N_{im}^m + 1)^{\frac{1}{2-\alpha}}, \]
and the following operating profits;
\[ \hat{\pi}_m = \frac{YA_{[1],m}(N_{im}^{m} + \alpha)^{1-\alpha}(1 - \alpha)}{w^{1-\alpha}(N_{im}^{m} + 1)^{1-\alpha}}. \]

The price in a market with imitators can now be expressed as;
\[ \hat{P}_m = w \left( \frac{N_{im}^{m} + 1}{N_{im}^{m} + \alpha} \right), \]
which reduces to equation (1) if the number of imitators is zero.

The value of introducing an improved product into market \( m \) in period \( t \) can now be expressed as;
\[
V_{s,m,t} = \frac{-Y_t \cdot \Psi_{m,t-2} \cdot c_s \cdot \rho^\theta}{L} + \frac{1}{N_{inn}^m} \left( E(\pi_{[1],m,t}) + \frac{E(\hat{\pi}_{m,t+1})}{1+r} \right) + \frac{(1 - \lambda)}{N_{inn}^m} \left( E(\pi_{m,t+2}) + \frac{E(\hat{\pi}_{m,t+4})}{1+r} \right) + \ldots,
\]
where \( c_s \) is the fixed cost of introducing an improved product for a new sequential innovator in the absence of patent protection, \( c'_s \) the same for the incumbent, \( \rho^\theta \) the factor difference in fixed costs due to protection, and \( 1/N_{inn}^m \) the probability of having the highest-quality product.

The lack of a time subscript on \( r \) and \( N_{inn}^m \) reflects the fact that these variables will be constant along the BGP. The expectations operator on current and future profits reflects the fact that an innovator is uncertain about its quality draw. The presence of future rounds of innovation in equation (2) accounts for the probability \( (N_{inn}^m)^{k/2} \) that the firm with the highest quality in period \( t \) will repeatedly draw the highest quality every two periods up to and including in period \( t + k \).
The value of creating an original product at time $t$ can be expressed as:

$$V_{o,m,t} = -\frac{Y_t}{L} \cdot c_o \cdot \rho^\phi + \mathbb{E}(\pi_{[1,m,t]}) + \frac{\mathbb{E}(\hat{\pi}_{m,t+1})}{1 + r}.$$ 

$$-\frac{Y_{t+2}}{L} \cdot \frac{\Psi_{m,t} \cdot c_s'(1 - \lambda)}{(1 + r)^2} + \frac{(1 - \lambda)}{(1 + r)^2 N_{inn}^m} \left( \mathbb{E}(\pi_{[1,m,t+2]}) + \frac{\mathbb{E}(\hat{\pi}_{m,t+3})}{1 + r} \right)$$

$$-\frac{Y_{t+4}}{L} \cdot \frac{\Psi_{m,t+2} \cdot c_s'(1 - \lambda)^2}{(1 + r)^4 N_{inn}^m} + \frac{(1 - \lambda)^2}{(1 + r)^4 (N_{inn}^m)^2} \left( \mathbb{E}(\pi_{[1,m,t+4]}) + \frac{\mathbb{E}(\hat{\pi}_{m,t+5})}{1 + r} \right) + \ldots$$

An imitator that enters a market with a copy of an incumbent’s product can expect operating profits for only one period. The value of introducing an imitation into market $m$ in period $t$ can be expressed as:

$$\hat{V}_{m,t} = -\frac{Y_t}{L} \cdot \Psi_{m,t-1} \cdot \hat{c} \cdot \rho^\gamma + \hat{\pi}_{m,t},$$

where $\hat{c}$ is the fixed cost of introducing an imitation in the absence of patent protection, and $\rho^\gamma$ the factor difference in fixed costs due to protection.

Finally, I assume each firm operates in only one product market, thus allowing the number of firms in the economy to be pinned down.

### 2.4.4 Stationary Decentralized Equilibrium

A stationary decentralized equilibrium is a constant number of innovators per market $N_{inn}^m$, imitators per market $N_{im}^m$, product markets $M$, and markups over marginal cost before imitation $P/w$, and after imitation $\hat{P}/w$, as well as a constant growth rate $g$ of final-good output $Y$, and the wage rate $w$, such that the following conditions are satisfied:

(i) Consumer Optimization: $r = \frac{1+g}{\beta} - 1$

(ii) Final-Good Firm Optimization: $P_m = \left(\frac{Y_A_{[1,m]}}{y_{[1,m]}}\right)^{1-\alpha}$ and $\hat{P}_m = \left(\frac{Y_{A_{[1,m]}}}{y_m(1+N_{inn}^m)}\right)^{1-\alpha}$
(iii) Intermediate Producer Optimization: \( \frac{P_m}{w} = \frac{1}{\alpha} \) and \( \frac{\hat{P}_m}{w} = \frac{N_m^{im} + 1}{N_m^{im} + \alpha} \)

(iv) Free Entry: \( V^o = V^s_m = \hat{V}_m = 0 \)

(v) Market Clearing (Goods): \( Y = \left( \int_0^M A_{[1],m}^{1-\alpha} y_{[1],m} dm \right)^{\frac{1}{\alpha}} \)

(vi) Market Clearing (Labor): \( L = \int_0^M y_{[1],m} dm \)

where conditions (ii) through (iv) are understood to hold for all \( m \in M \).

To solve for the stationary equilibrium I start with the market clearing conditions. In each period, half of all markets feature innovations and half feature identical competitors. Continue to let \( M^{inn} \subset M \) denote the subset of product markets in which innovation occurs in the current period, indexed by \( m^{inn} \), and let \( M^{im} \subset M \) denote the subset of product markets in which imitation occurs in the current period, indexed by \( m^{im} \). Substituting the optimal output of each producer (\( y_{[1],m^{inn}} \) and \( \hat{y}_{m^{im}} \)) results in the following expressions;

\[
\frac{w}{\alpha} = M^{\frac{1-\alpha}{\alpha}} \left[ L \right]^{\frac{1-\alpha}{\alpha}} \quad \text{and} \quad \frac{Y}{\alpha} = M^{\frac{1-\alpha}{\alpha}} \left[ \frac{L}{\alpha} \right],
\]

where \([L]\) \( \equiv \frac{1}{2} \cdot \left[ \alpha^{\frac{1-\alpha}{\alpha}} A_{[1]}^{inn} + \frac{N^{im} + \alpha}{N^{im} + 1} \right]^{\frac{1-\alpha}{\alpha}} A_{[1]}^{im} \),

\([\alpha]\) \( \equiv \frac{1}{2} \cdot \left[ \alpha^{\frac{1-\alpha}{\alpha}} A_{[1]}^{inn} + \frac{N^{im} + \alpha}{N^{im} + 1} \right]^{\frac{1-\alpha}{\alpha}} A_{[1]}^{im} \),

\( \bar{A}_{[1]} = M_k \int_0^M A_{[1],m} dm^k, \text{ } k \in \{inn, im\} \),

and the quality of original products \( A_0 \) is understood to be equal to \( A_{[1]}^{inn} \).

Given the assumptions made about each market’s best quality in period zero, \( A_{[1],t}^{inn} = A_{[1],t}^{im} \cdot (1 + \mathbb{E}[h_{[1]}])^{t/2} \) and \( A_{[1],t}^{im} = (1 + \mathbb{E}[h_{[1]}])^{t/2} \), for all \( t \). The wage and aggregate output per
capita can therefore be expressed as:

\[ w = M^{\frac{1-\alpha}{\alpha}} [L]^{\frac{1-\alpha}{\alpha}} (A_{[1]}^{im})^{\frac{1-\alpha}{\alpha}} \quad \text{and} \quad \frac{Y}{L} = M^{\frac{1-\alpha}{\alpha}} [L]^{\frac{1}{\alpha}} (A_{[1]}^{im})^{\frac{1-\alpha}{\alpha}}, \]

where \([L] \equiv \frac{1}{2} \cdot \left[ \alpha^{\frac{1-\alpha}{\alpha}} (1 + \mathbb{E} [h_{[1]}])^{1/2} + \left( \frac{N_{im} + \alpha}{N_{im} + 1} \right)^{\frac{1-\alpha}{\alpha}} \right],\]

and \([Y] \equiv \frac{1}{2} \cdot \left[ \alpha^{\frac{1-\alpha}{\alpha}} (1 + \mathbb{E} [h_{[1]}])^{1/2} + \left( \frac{N_{im} + \alpha}{N_{im} + 1} \right)^{\frac{1-\alpha}{\alpha}} \right].\]

Both the measure of product markets \(M\) and the expected value of the best innovation \(\mathbb{E} [h_{[1]}]\) in each period are constant, so the growth rate \(g\) depends only on \(\mathbb{E} [h_{[1]}]\), which in turn depends on the number of sequential innovators per market \(N_{inn}\):

\[ 1 + g = \frac{Y_t}{Y_{t-1}} = \left( \frac{A_{[1],t}^{im}}{A_{[1],t-1}^{im}} \right)^{\frac{1-\alpha}{\alpha}} = (1 + \mathbb{E} [h_{[1]}])^{\frac{1-\alpha}{2\alpha}}. \]

With expressions for \(Y, w,\) and \(g\) in hand, the equilibrium number of product markets \(M\), sequential innovators \(N_{inn}\), and imitators \(N_{im}\) can be solved for implicitly using the free-entry conditions (iv):

\begin{align*}
\text{imitators:} \quad \hat{c}\rho^\gamma &= \frac{L(1 - \alpha)(N_{im} + \alpha)^{\frac{\alpha}{1-\alpha}}}{M[L](N_{im} + 1)^{\frac{2-\alpha}{2-\alpha}}}, \\
\text{sequential:} \quad c_s\beta^\theta + \frac{c_s'\beta^2(1 - \lambda)}{[N_{inn} - \beta^2(1 - \lambda)]} &= \frac{L(1 - \alpha)}{M[L][N_{inn} - \beta^2(1 - \lambda)]} \cdot \left[ \alpha^{\frac{\alpha}{1-\alpha}} (1 + g)^{\frac{\alpha}{1-\alpha}} + \frac{\beta(N_{im} + \alpha)^{\frac{\alpha}{1-\alpha}}}{(N_{im} + 1)^{\frac{2-\alpha}{2-\alpha}}} \right],
\end{align*}
original: \( c_\alpha \rho^\phi + \frac{c'_s \beta^2 (1 - \lambda) N^{inn}}{[N^{inn} - \beta^2 (1 - \lambda)]} \) (5)

\[
\frac{L(1 - \alpha)N^{inn}}{M[L][N^{inn} - \beta^2 (1 - \lambda)]} \cdot \left[ \frac{\alpha}{\alpha - \sigma} (1 + g) \frac{\alpha}{\alpha - \sigma} + \frac{\beta (N^{im} + \alpha)^{\frac{\alpha}{\alpha - \sigma}}}{(N^{im} + 1)^{\frac{\alpha}{\alpha - \sigma}}} \right].
\]

Given some distribution for \( h_i \), the random variable determining each innovator’s quality, all other variables are functions of \( N^{im}, N^{inn}, \) and \( M \).

### 2.5 Theoretical Results

Here I examine how the stationary equilibrium of the economy depends on a number of exogenous variables. I start by discussing the effects of patent protection.

#### 2.5.1 Patent Protection

By combining the free entry conditions for sequential and original innovators (equations 4 and 5), I can solve for the number of sequential innovators per market \( N^{inn} \);

\[
N^{inn} = \left( \frac{c_\alpha}{c_s} \right) \rho^{\phi - \theta}.
\] (6)

Notice that equation 6 contains no trace of imitation. To the extent that patent protection affects imitation, this changes the value of entry equally for both original and sequential innovators. An increase (decrease) in imitation will reduce (increase) the number of markets \( M \), via a decrease (increase) in the number of original innovators. This decrease (increase) in the number of markets will increase (decrease) the size of each market just enough to leave the value of entry for sequential innovators unchanged. As a result, the aggregate number of

---

8The independence of sequential innovation from the level of imitation does not hold for all assumptions about market structure. I discuss the implications of relaxing my assumptions about market structure in Section ??.
sequential innovators (which is proportional to $M^{inn}$) will decrease (increase), but $N^{inn}$ (and therefore growth) will remain constant. If patent protection affects the costs of both original and sequential innovators in the same proportion ($\phi = \theta$), then both $N^{inn}$ and the long-run growth rate of the economy are independent of the strength of patent protection. In this case, patent protection affects the value of entry by the same factor for both original and sequential innovators. Using the same intuition as above, any change in the value of entry for sequential innovators in this case will be fully offset by a change in the number of original innovators. If patent protection (for example) increases the cost of sequential innovation more than original innovation ($\theta > \phi$) while leaving imitation unchanged, then both the number of sequential innovations per market and the long-run growth rate will decrease, as in Chu, Cozzi, and Galli (2012). If $\theta < \phi$, then the opposite will be true.

Regardless of whether the strength of patent protection affects the growth rate along the BGP, it can still affect the level of aggregate output. To understand how, consider first the case where the factor difference in costs due to protection are the same for both original innovators and imitators ($\gamma = \phi$), and assume for the moment that $N^{inn}$ is fixed. The free entry condition for original innovators (5) shows that any increase in protection $\rho$ will be exactly offset by a change in the number of markets $M$, given some fixed number of imitators $N^{im}$. If $\rho^\phi \cdot M$ is constant and equal to $\rho^\gamma \cdot M$, then the free entry condition for imitators (3) implies that $N^{im}$ will indeed stay constant. An increase in patent protection must therefore increase (if $\phi = \gamma < 0$) or reduce (if $\phi = \gamma > 0$) the number of markets, the number of firms, aggregate output, and welfare, while leaving unchanged the long-run growth rate, the number of imitators, and markups. The intuition for why protection does not affect imitation here is similar to that above. An increase (decrease) in the cost of original innovation due to patent protection results in a lower (greater) equilibrium number of markets. This increases (decreases) the size of each market just enough to offset the higher (lower) cost of imitation (when $\gamma = \phi$), so that the incentive to imitate is left unchanged. If $\phi \neq \theta$ then $N^{inn}$ and the growth rate will change (as explained above), moving the number of imitators $N^{im}$ in the opposite direction.
If \( \gamma \) is greater than \( \phi \), that is, if patent protection increases costs for imitators more than for original innovators (or decreases them less), then the number of imitators per market \( N^{im} \) is decreasing in the strength of patent protection \( \rho \) (for a given \( N^{inn} \)). As a result, markups are higher, which increases the return to innovation (the last terms in equations 4 and 5). If \( \gamma > \phi > 0 \) then this decrease in imitation will at least partially offset the additional costs of innovation, so that the net effect of stronger patent protection on original innovation is ambiguous. In this case, protection is more likely to increase original innovation the higher is \( \gamma \). An increase in innovation is also more likely if the number of imitators \( N^{im} \) is already low (say, because of a high cost of imitation \( \hat{c} \)). When \( N^{im} \) is already low, a further decrease has a large effect on both markups and on the patent holder’s market share.

### 2.5.2 Other

It is clear from the free entry conditions (equations 3 through 5) that the number of product markets \( M \) is linear in population \( L \). A larger population does not increase the size of each market, and so the number of imitators, markups, growth, and the number of firms per worker are all independent of population. This is in stark contrast to every other model with endogenous markups, but consistent with the empirical evidence documented by Bento and Restuccia (2016).

A higher discount rate \( \beta \) is associated with more innovation, and therefore a larger number of markets \( M \). A higher \( M \) shrinks the size of each market, so imitation is reduced. As a result, markups under imitation are higher. The growth rate is independent of \( \beta \) (equation 6), as the change in \( M \) leaves the value of entry for sequential innovators unchanged. Note that these results do not imply that subsidies or taxes need affect markups. For example if a proportional tax were applied equally to the costs of innovators and imitators, the result would be the same as an increase in \( \rho \) when \( \gamma = \phi = \theta = 1 \). \( M \) would decrease, but markups would remain constant.
3 Quantitative Analysis

In this section I quantify the impact of patent protection on an economy. I begin by interpreting the data while maintaining the assumptions made about market structure in Section 2. I then discuss the implications of alternative assumptions.

3.1 Benchmark

The most important parameters required for this analysis are $\phi$, $\theta$, and $\gamma$, which determine the relative cost effects associated with patent protection for sequential innovators, original innovators, and imitators. Importantly, the signs and ordinal rankings of $\phi$, $\theta$, and $\gamma$ are associated with different effects of protection on markups, the number of firms in the economy, and the fraction of innovations that are original rather than sequential. Consider first the value of $\phi$, relative to $\theta$. If $\phi = \theta$ then patent protection increases the costs of both sequential and original innovators proportionately, leaving the number of sequential innovators $N_{inn}$ unchanged. If $\phi > \theta$ then $N_{inn}$ will increase with stronger protection, and vice versa. In the model, the ratio of original to total innovations is:

$$\frac{\# \text{ original}}{\# \text{ total innovations}} = \frac{\lambda}{\lambda + (1 - \lambda)N_{inn}},$$

where $\lambda/2$ is the fraction of markets with original innovations, $(1 - \lambda)/2$ the fraction with sequential innovations, and $N_{inn}$ the number of sequential innovations per market. This ratio depends solely on $N_{inn}$ (as well as exogenous parameters), and the effect of patent protection on $N_{inn}$ depends solely on $\phi$. To obtain a value for $\phi$ I look to cross-country panel data on the strength of patent protection and on the fraction of patents that are original. If this measure of originality is unrelated to patent protection, then I must conclude that $\phi = \theta$. If originality is increasing in patent protection, then the estimated coefficient on patent protection can be used
along with other calibration targets to back out an implied value for $\phi < \theta$ (and vice versa). As I detail below, it turns out that the data is consistent with $\phi$ equal to $\theta$.

As a measure of the strength of patent protection I use the Ginarte-Park Patent Rights Index developed in Ginarte and Park (1997) and updated in Park (2008). Each country-year in the Index is given a continuous value from zero to five, with the index value increasing in coverage (the number of product-types protected), duration of protection, the scope of patent rights, and membership in international intellectual property treaties, while decreasing in the number of restrictions on patent rights (like compulsory licensing). The Index reports values for each country every five years from as early as 1960 to 2005.

For a measure of the fraction of patents that are original I use data from the NBER Patent Database (Hall, Jaffe, and Trajtenberg, 2001). The NBER database contains a measure of originality for each utility patent granted in the U.S. from 1973 to 1999. For each patent, data is available for country of origin, industry, and year of application. This measure of originality is constructed in such a way as to be higher when a patent application cites previous patents across a wide range of fields, and lower when it cites previous patents mainly within its own field. To construct a measure of the fraction of patents that are original for each country-industry-year, I first calculate a threshold equal to the median originality value across patents originating from the U.S. in the same industry-year. I then construct my measure to be equal to the fraction of patents from a country-industry-year that have originality values higher than this threshold. Using originality scores in U.S. industry-years as benchmarks for each industry controls for differences in originality across industries and over time that are unrelated to the strength of patent protection in the originating countries.

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9 The Ginarte-Park Index has been used widely. Some examples are Smith (2001), Branstetter, Fisman, and Foley (2006), and Antrás, Desai, and Foley (2009).

10 To make the NBER data comparable with the Ginarte-Park Index, I pool observations for each country-industry across five-year intervals. For example, the measure of originality associated with the Patents Rights Index value for 1990 is constructed using patent data from 1988 to 1992. The word “year” refers to this five-year interval.

11 Given this arbitrary threshold, my measure can not reliably measure the actual ratio of original to sequential innovations. What is important here, though, is that it captures the change in the fraction of original innovations over time.
Table 1: Patent Protection and Originality

<table>
<thead>
<tr>
<th>dependent variable: fraction of original patents</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patent Index</td>
<td>-0.08</td>
<td>-0.09</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
<td>(0.05)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Openness</td>
<td>-0.21***</td>
<td>-0.11**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.06)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heritage Index</td>
<td>1.22***</td>
<td>0.91***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td>(0.21)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entry Barriers</td>
<td>0.28***</td>
<td>0.31***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance</td>
<td>0.10***</td>
<td>0.06**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

adjusted-$R^2$ 0.19 0.20 0.14 0.20
observations 4462 4264 4462 4264

Notes: Originality of patents is from Hall, Jaffe, and Trajtenberg (2001), Patent Index is from Ginarte and Park (1997) and Park (2008), Openness is from Penn World Tables v8, Heritage Index is from Heritage Foundation (2015), and Entry Barriers is from World Bank (2014). All regressions include time dummies, and Patent Index is relative to the U.S. Robust standard errors in parentheses. *** and ** refer to one and five percent levels of significance.

Column 1 of Table 1 reports the result of an OLS regression of the ratio of original to total patents on the strength of patent protection. In this regression (and all others reported in Table 1) I include time dummies to control for time effects not related to variation in patent protection, as well as country-industry dummies to control for fixed country-specific factors that may affect the fraction of patents which are original. Identification here is therefore coming solely from variation in the strength of patent protection over time within a country, relative to other countries. The coefficient on patent protection is negative, but statistically insignificant. This insignificance result is robust to the addition of other variables that could plausibly help explain the ratio of original to total patents. Column 2 reports the OLS estimates for patent protection, openness to trade, institutional quality (Heritage Index), and a measure of entry barriers.\(^{12}\) In this regression I also control for the distance of a country’s capital city from

\(^{12}\)Openess to trade is measured as imports plus exports as a fraction of GDP for each year covered by the Patent Index, reported in the Penn World Tables v8. Heritage Index is a measure of economic freedom, reported
Washington D.C. This helps control for the possibility that a) original patents tend to be of higher or lower quality than sequential patents, and b) greater distance from the U.S. raises the quality threshold necessary to make patenting an innovation in the U.S. worthwhile, thus disproportionately affecting the number of sequential and original patents applied for. The reported coefficient for patent protection is similar to that in Column 1 and still insignificant. Columns 3 and 4 report the estimates from analogous regressions using analytic weights equal to the number of patents used to construct the originality measure for each country-industry-year. The estimated coefficients on patent protection are again insignificantly different from zero.

Using the median score of originality for U.S. patents as the threshold for each industry-year is somewhat arbitrary, so for robustness I repeat the above analysis using a threshold equal to the mean U.S. value, as well as directly using the originality measure from Hall, Jaffe, and Trajtenberg (2001) as a dependent variable. Although not shown here, the estimated coefficient on patent protection in all cases is insignificant. I also repeat the regressions described above using lagged values for the strength of patent protection, and again using only country fixed effects rather than country-industry. Again, the estimated coefficient on patent protection in each regression is insignificant. Given these results, I conclude that patent protection affects the costs of original and sequential innovation proportionately. For the purposes of calibration, I therefore assume \( \phi = \theta \).

I now turn to \( \gamma \), which determines the impact of patent protection on the cost of imitation. If \( \gamma = \phi = \theta \), then patent protection affects the costs of innovation and imitation proportionately, while \( \gamma > \phi = \theta \) implies that protection increases the cost of imitation more than the cost of innovation (or decreases it less) and \( \gamma < \phi = \theta \) implies the opposite. All else equal, a higher \( \gamma \) therefore implies a larger decrease (or smaller increase) in the number of imitators per market and a larger increase (or smaller decrease) in average markups. To obtain a value for \( \gamma \) I look for 1995. Entry barriers are proxied by the regulatory cost of starting a formal business, as measured by the World Bank’s Doing Business Surveys. For more details about these data see Feenstra, Inklaar, and Timmer (2015), Heritage Foundation (2015), and World Bank (2014).
to cross-country data on the strength of patent protection and on markups. If markups are unrelated to patent protection, then I must conclude that $\gamma = \phi = \theta$. If markups are increasing in patent protection, then the estimated coefficient on patent protection can be used along with other calibrated parameters in the model to back out a higher implied value for $\gamma$ (and vice versa). As I detail below, it turns out that the data is consistent with $\gamma = \phi = \theta$.

As a measure of patent protection I continue to use the Ginarte-Park Patent Rights Index (Ginarte and Park, 1997, and Park, 2008). I construct my measure of markups using data from the World Bank’s Enterprise Surveys. The Enterprise Surveys contain establishment-level data on sales, intermediate purchases, and inputs, and are constructed in such a way as to be comparable across countries and representative of establishments with at least five workers. The World Bank has constructed two separate datasets, one with data for 2002-2005 and a second with data for 2006-2014. The earlier dataset contains data for 45 (usable) countries and up to 16 manufacturing industries, while the later dataset contains 77 countries and 11 industries. For each country-industry I calculate markups as the ratio of total value-added to the total wage bill.\footnote{This measure gives more weight to the markups of larger establishments.} I neglect capital costs in my construction of markups for two reasons. First, including capital necessitates an adjustment for the large differences in the nominal rental rate across countries (Feenstra, Inklaar, and Timmer, 2015), while the ratio of value-added to the wage bill is robust to these differences. Although the calculated markups for all countries will be biased upward due to the omission of capital costs, this measure should still capture the differences in markups due to differences in patent protection. Second, several countries and many establishments within each country do not report capital, so requiring capital data would significantly reduce the number of usable countries and increase the variance of my measure across countries. Using only data from establishments with at least five workers mitigates the mismeasurement of the wage bill due to the fact that in poorer countries many small establishments employ unpaid labor and do not report the implied wages of own-account workers (Gollin, 2002; Bento and Restuccia, 2016). That establishments in the data are classified by
industry allows me to control for differences in industry composition across countries.

Table 2: PATENT PROTECTION AND MARKUPS

<table>
<thead>
<tr>
<th>dependent variable: markups</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td>Patent Index</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Openness</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Heritage Index</td>
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<tr>
<td></td>
</tr>
<tr>
<td>Entry Barriers</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>adjusted-R²</td>
</tr>
<tr>
<td>observations</td>
</tr>
</tbody>
</table>

Notes: All variables in logs. Markup data is from the World Bank’s Enterprise Surveys. Patent Index is from Park (2008), Openness is from Penn World Tables v8, Heritage Index is from Heritage Foundation (2015), and Entry Barriers is from World Bank (2014). All regressions include industry dummies. Robust standard errors in parentheses. ***, **, and * refer to one, five, and ten percent levels of significance.

Table 2 shows the results of six regressions of markups on patent protection, three each using markup data from 2002-2005 and from 2006-2014. The first column of each (Columns 1 and 4) suggest that markups are unrelated to patent protection, except for the coefficient estimate from the most recent World Bank data. In the second columns (2 and 5) I include a measure of each country’s openness to trade (described above) to address the fact that the Ginarte and Park Patent Rights Index allocates a higher index value to countries ratifying international property rights treaties. Ratification of these treaties is often accompanied by, and even a condition for, entry into the World Trade Organization or other trade agreements which could themselves impact competition and markups. To otherwise control for differences in the quality of institutions and policies across countries, I include the Heritage Foundation’s (2015) Economic Freedom Index and the World Bank’s (2014) measure of entry barriers. Controlling

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14For the earlier sample, I use the reported Patent Index value for 2000. For the later sample, I use the value for 2005 (the most recent value reported in Park (2008)).
for these other variables results in insignificant coefficient estimates for patent protection (p-values all greater than 10 percent), even in the most recent data (Column 5). Column 3 and 6 repeat the regressions from 2 and 4 using the number of observations included in my measure of markups as analytic weights for each country-industry. Again, the coefficient estimates for patent protection are insignificantly different from zero.

The model in Section 2 suggests that any effect on markups should be greater in industries where markups are otherwise high. For robustness, I therefore repeat the regressions from Table 2 using only industries with high markups.\(^{15}\) In all of these regressions but one, the estimated coefficient on patent protection is insignificant. The sole exception is the analogous regression to Column 4 where the estimated coefficient remains significant, although only at the 10 percent level.

The results from Tables 1 and 2 suggest that \(\gamma = \phi = \theta\), that is, patent protection affects the costs of imitators, original innovators, and sequential innovators proportionately. As discussed in Section 2.5.1 above, this implies that patent protection affects only the measure of markets \(M\) in steady state, and not the growth rate or the number of imitators. The factor difference in output per capita between an economy with weak protection \(\rho_L\) and strong protection \(\rho_H\) can therefore be expressed simply as:

\[
\frac{Y(\rho_L)}{Y(\rho_H)} = \left( \frac{M(\rho_L)}{M(\rho_H)} \right)^{\frac{1-\alpha}{\alpha}}.
\]

Whether stronger patent protection increases or decreases aggregate innovation and the number of product markets depends on whether protection increases or decreases the costs of innovators and imitators, that is, whether \(\gamma = \phi = \theta < 0\) or \(\gamma = \phi = \theta > 0\). The Ginarte-Park Index is an index of the strength of patent protection, so it is not obvious how different index values can be mapped to \(\rho\) in the model. But one of the implications of \(\gamma = \phi = \theta\) is that differences in

\(^{15}\)I calculate the cross-country average markup for each industry, and then use industries with average markups above the median for each sample.
number of firms (per worker) across countries caused by differences in patent protection imply proportional differences in the number of product markets $M$ (per worker). In the model the number of firms is;

$$
\frac{M}{2} \left[ \lambda + (1 - \lambda)N^{inn} + 1 + N^{im} \right],
$$

where $M \cdot 1/2 \cdot \lambda$ is both the number of dying markets and the number of original innovators, $M \cdot 1/2 \cdot (1 - \lambda)$ the number of surviving markets with sequential innovations, $N^{inn}$ the number of sequential innovators per surviving market, $M \cdot 1/2$ the number of markets with imitators, and $1 + N^{im}$ the number of firms competing in each market with imitators. If $\gamma = \phi = \theta > 0$ then differences in patent protection $\rho$ leave $N^{inn}$ and $N^{im}$ unchanged. Any factor difference in the number of firms (per worker) between two countries due to a difference in patent protection can therefore be interpreted as a factor difference in the number of markets $M$ (per worker).

To estimate the relationship between patent protection and the number of firms per worker I use data from Bento and Restuccia (2016), which reports the average number of workers per manufacturing establishment for 134 economies. Figure 1 plots the number of establishments per worker against the Ginarte-Park Index for 80 countries, and shows a clear negative relationship between the strength of patent protection and the number of establishments, which implies $\gamma = \phi = \theta > 0$. Column 1 of Table 3 reports that the coefficient estimate from a regression of (logged) establishments per worker on the (logged) patent rights index value is equal to -1.64 and significant at the one percent level.

The number of establishments per worker can obviously differ across countries for reasons other than patent protection, so in Column 2 of Table 3 I report the results of a regression of establishments per worker on patent protection, openness to trade, and a measure of institutional quality. The coefficient estimate on the Ginarte-Park Index is -0.69, implying about two-thirds of a percent drop in the number of firms from a one percent increase in the Index value. To quantify the effect on aggregate productivity from patent protection in the U.S., I first esti-

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16 As my measure of establishments per worker for each country, I use the inverse of workers per establishment reported in Bento and Restuccia (2016).
mate the increase in the number of firms that would result if the U.S. reduced the strength of patent protection to the average level observed in the bottom decile of countries included in Column 2. For 2005, Park (2008) reports a value of 488 for the U.S. and an average value of 213 for the eight countries with the weakest protection. An elasticity of establishments with respect to protection of -0.69 implies that reducing protection would increase the number of establishments in the U.S. by a factor of \((213/488)^{-0.69} = 1.77\). This implies an increase in aggregate productivity by a factor of \(177^{1/(1-\alpha)}\), where \(1/(1-\alpha)\) is the elasticity of substitution between product markets. Using a value of 5.5/6.5 for \(\alpha\) (Imbs and Méjean, 2015), this implies that weakening patent protection in the U.S. would lead to a permanent increase in aggregate output by 9 percent.\(^{17}\)

3.2 Alternative Market Structures

In principle, alternative assumptions about market structure in Section 2 could change the predicted effects of patent protection. In this section I argue that the quantitative results

\(^{17}\)Imbs and Méjean (2015) suggest an elasticity of substitution of 6.5 for macroeconomic models that assume only one industry. This implies a value for \(\alpha\) of 5.5/6.5.
Table 3: Number of Establishments and Patent Protection

<table>
<thead>
<tr>
<th>dependent variable: manufacturing establishments per worker</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patent Index</td>
<td>-1.64***</td>
<td>-0.69**</td>
</tr>
<tr>
<td></td>
<td>(0.26)</td>
<td>(0.31)</td>
</tr>
<tr>
<td>Openness</td>
<td>-0.21**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td></td>
</tr>
<tr>
<td>Heritage Index</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.49)</td>
<td></td>
</tr>
<tr>
<td>Entry Barriers</td>
<td>0.18***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.32</td>
<td>0.44</td>
</tr>
<tr>
<td>observations</td>
<td>80</td>
<td>75</td>
</tr>
</tbody>
</table>

Notes: All variables in logs. Establishments per worker is from Bento and Restuccia (2016). Patent Index is from Park (2008), Openness is from Penn World Tables v8, Heritage Index is from Heritage Foundation (2015), and entry barriers is from World Bank (2014). Robust standard errors in parentheses. ***, **, and * refer to one, five, and ten percent levels of significance.

reported above are nonetheless robust to reasonable alternatives. Importantly, I continue to assume throughout this discussion that the number of product markets is large enough to make the markups of original innovators independent of the number of markets. This ensures that the number of firms per worker in the economy does not depend on the population. If the markups of original innovators were instead allowed to decrease (for example) in the number of markets then more populous economies would be predicted to have lower markups and fewer firms per worker (as in all current models with free entry and endogenous markups), which contradicts the finding in Bento and Restuccia (2016) that the number of firms per worker is independent of population.

If markups under imitation were assumed to be independent of the number of imitators, then the empirical relationship (or lack thereof) between protection and markups would provide no information about imitation. The quantitative results in Section 2 do therefore depend on the assumption that the number of imitators affects markups, although the exact relationship between imitators and markups is not important. This required assumption here is consistent
with evidence that markups generally change (in particular, decrease) after imitators enter a market (for example, Grabowski and Vernon, 1992).

Given the assumptions that the number of markets is large and markups under imitation depend on the number of imitators, it follows that the market structure for sequential innovators is irrelevant for the quantitative results. The lack of any relationship between the strength of patent protection and the fraction of original innovators implies that the number of sequential innovators per market is independent of patent protection. This must be because $\phi = \theta = \gamma$, as argued above. An alternative explanation is that stronger patent protection reduces the number of imitators, thus increasing markups under imitation. In such a scenario, $\phi = \theta$ implies that the number of sequential innovators $N^{inn}$ does not change if markups for sequential innovators are equal to markups for original innovators. But if markups for sequential innovators are lower than those for original innovators, then less imitation increases the value of entry for sequential innovators more than for original innovators. To maintain a constant $N^{inn}$, it must therefore be the case that $\theta > \phi$, so that the disproportionately higher costs for sequential innovators are such that the value of entry for sequential innovators is on net unaffected. But if $N^{inn}$ does not change, then markups under sequential innovation do not change, regardless of market structure. In this scenario, the model would therefore predict an increase in average markups due to patent protection, which is inconsistent with the lack of any empirical relationship reported above in Section 3.1.

As long as the number of product markets is assumed to be large and markups under imitation are assumed to depend on the number of imitators, then the quantitative results in Section 3.1 are maintained regardless of the particular market structures assumed for sequential innovators and imitators. Given the empirical findings above, it must be the case that patent protection does not affect markups or the number of sequential innovators per market, but does decrease the number of firms and therefore the number of product markets. Given the production function of the final good firm, the quantitative impact of a lower number of product markets
is determined solely by the elasticity of substitution between markets.

4 Conclusion

In this paper I develop a general equilibrium model that can generate any qualitative effect of patent protection on markups, imitation, long-run growth, and aggregate productivity. Disciplining the model parameters to match the relationships between these variables found in the data, I am able to quantitatively assess the impact of reducing the strength of patent protection from its current level in the United States. The results are surprising, but are clearly implied by the data: no change in markups, imitation, or long-run growth, but a significant increase in the number of firms and in aggregate productivity.

It is important to note that the analysis in this paper takes an aggregate view of the effects of patent protection. To the extent that the relative costs imposed by patent protection may differ across industries, different industries may react in different ways to protection. This implies that different levels of protection may be optimal across industries. But given the one-size-fits-all nature of current policy, the present model interprets the data as suggesting that substantial net gains in productivity can be achieved by weakening patent protection.
References


